Topics in Network Utility Maximization: Interior Point and Finite-step Methods

Abstract: Network utility maximization has emerged as a powerful tool in studying flow control, resource allocation and other cross-layer optimization problems. In this work, we study a flow control problem in the optimization framework. The objective is to maximize the sum utility of the users subject to the flow constraints of the network. The utility maximization is solved in a distributed setting; the network operator does not know the user utility functions and the users know neither the rate choices of other users nor the flow constraints of the network.

We build upon a popular decomposition technique proposed by Kelly [Eur. Trans. Telecommun., 8(1), 1997] to solve the utility maximization problem in the aforementioned distributed setting. The technique decomposes the utility maximization problem into a user problem, solved by each user and a network problem solved by the network. We propose an iterative algorithm based on this decomposition technique. In each iteration, the users communicate to the network their willingness to pay for the network resources. The network allocates rates in a proportionally fair manner based on the prices communicated by the users. The new feature of the proposed algorithm is that the rates allocated by the network remains feasible at all times. We show that the iterates put out by the algorithm asymptotically tracks a differential inclusion. We also show that the solution to the differential inclusion converges to the system optimal point via Lyapunov theory. We use a popular benchmark algorithm due to Kelly et al. [J. of the Oper. Res. Soc., 49(3), 1998] that involves fast user updates coupled with slow network updates in the form of additive increase and multiplicative decrease of the user flows. The proposed algorithm may be viewed as one with fast user update and fast network update that keeps the iterates feasible at all times. Simulations suggest that our proposed algorithm converges faster than the aforementioned benchmark algorithm.

When the flows originate or terminate at a single node, the network problem is the maximization of a so-called d-separable objective function over the bases of a polymatroid. The solution is the lexicographically optimal base of the polymatroid. We map the problem of finding the lexicographically optimal base of a polymatroid to the geometrical problem of finding the concave cover of a set of

points on a two-dimensional plane. We also describe an algorithm that finds the concave cover in linear time.

Next, we consider the minimization of a more general objective function, i.e., a separable convex function, over the bases of a polymatroid with a special structure. We propose a novel decomposition algorithm and show the proof of correctness and optimality of the algorithm via the theory of polymatroids. Further, motivated by the need to handle piece-wise linear concave utility functions, we extend the decomposition algorithm to handle the case when the separable convex functions are not continuously differentiable or not strictly convex. We then provide a proof of its correctness and optimality.